DISPERSION REPRESENTATIONS FOR HARD EXCLUSIVE REACTIONS [a]

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Abstract

A number of hard exclusive scattering processes can be described in terms of generalized parton distributions (GPDs) and perturbative hard-scattering kernels. Both the physical amplitude and the hard-scattering kernels fulfill dispersion relations. We show that their consistency at all orders in perturbation theory is guaranteed if the GPDs satisfy certain integral relations. These relations are fulfilled thanks to Lorentz invariance.

1 Introduction

For hard exclusive processes that can be calculated using collinear factorization, one may write down dispersion relations both for the physical process and for the parton-level subprocess. The question of consistency between both representations turns out to be nontrivial. Important progress has recently been reported in [1], where it was shown that this consistency is ensured by Lorentz invariance in the form of the polynomiality property for generalized parton distributions (GPDs). The studies in [1] were carried out using the Born-level approximation of the hard-scattering subprocess. In particular, they showed that to this accuracy not only the imaginary but also the real part of the process amplitude can be represented in terms of GPDs $F(x, \xi, t)$ along the line $x = \xi$ in the x–ξ plane. It is natural to ask how the situation changes when including radiative corrections to the hard-scattering kernel. Here we discuss dispersion representations for hard exclusive processes to all orders in perturbation theory, generalizing the leading-order results derived for the unpolarized quark GPDs in [1]. More details, as well as results for polarized quarks and for gluons, where special issues arise, can be found in our journal publication [2].

2 Dispersion relations

The exclusive processes we discuss here are deeply virtual Compton scattering (DVCS) and light meson production,

$$\gamma^*(q) + p(p) \to \gamma(q') + p(p') \,, \qquad \gamma^*(q) + p(p) \to M(q') + p(p') \,, \tag{1}$$

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where four-momenta are indicated in parentheses. Since the processes in (1) involve particles with nonzero spin, the appropriate quantities for discussing dispersion relations are invariant amplitudes, which have simple analyticity and crossing properties. An explicit decomposition for Compton scattering can be found in [3].

We use the Mandelstam variables $s = (p+q)^2$, $t = (p-p')^2$, $u = (p-q')^2$, and consider an invariant amplitude $\mathcal{F}^{[\sigma]}(\nu, t)$ with definite signature σ under $s \leftrightarrow u$ crossing,

$$\mathcal{F}^{[\sigma]}(-\nu, t) = \sigma \mathcal{F}^{[\sigma]}(\nu, t)\,, \tag{2}$$

where $2\nu = s - u$. At $t \leq 0$ the imaginary part of the amplitude is due to the s-channel discontinuity for $\nu > 0$ and to the u-channel discontinuity for $\nu < 0$. The fixed-t dispersion relation with one subtraction reads

$$\begin{aligned}
&\mathrm{Re}\, \mathcal{F}^{[\sigma]}(\nu, t) - \mathrm{Re}\, \mathcal{F}^{[\sigma]}(\nu_0, t) \\
&= \frac{1}{\pi} \int_{\nu_{th}}^{\infty} d\nu' \, \mathrm{Im}\, \mathcal{F}^{[\sigma]}(\nu', t) \left[\frac{1}{\nu' - \nu} + \sigma \frac{1}{\nu' + \nu} - \frac{1}{\nu' - \nu_0} - \sigma \frac{1}{\nu' + \nu_0} \right],
\end{aligned} \tag{3}$$

where ν_{th} is the value of ν at threshold. Its validity requires

$$\nu^{-2} \mathcal{F}^{[+]}(\nu, t) \to 0\,, \qquad \nu^{-1} \mathcal{F}^{[-]}(\nu, t) \to 0\,. \tag{4}$$

for $|\nu| \to \infty$. We consider dispersion relations for the processes (1) in the Bjorken limit of large $-q^2$ at fixed q^2/ν and t. It is useful to trade ν for the scaling variable

$$\xi = -\frac{(q + q')^2}{2(p + p') \cdot (q + q')} = -\frac{q^2}{s - u} = -\frac{q^2}{2\nu}\,, \tag{5}$$

where we have neglected q'^2 and t compared with q^2 in the numerator. The factorization theorems state that in the Bjorken limit certain invariant amplitudes become dominant and can be written as convolutions of partonic hard-scattering kernels with quark or gluon GPDs. We discuss the contribution of unpolarized quark distributions $F^q = \{H^q, E^q\}$ to the leading invariant amplitudes for DVCS or meson production,

$$\mathcal{F}^{q[\sigma]}(\xi, t) = \int_{-1}^{1} dx \, \frac{1}{\xi} C^{q[\sigma]}\left(\frac{x}{\xi}\right) F^q(x, \xi, t) \tag{6}$$

where for brevity we do not display the dependence of $\mathcal{F}^{q[\sigma]}$ and $C^{q[\sigma]}$ on q^2. The hard-scattering kernel satisfies the symmetry relation

$$C^{q[\sigma]}(-x/\xi) = -\sigma\, C^{q[\sigma]}(x/\xi)\,. \tag{7}$$

In the Bjorken limit the Mandelstam variables for the hard-scattering subprocess are

$$\hat{s} = xs + \tfrac{1}{2}(1 - x)q^2\,, \qquad \hat{u} = xu + \tfrac{1}{2}(1 - x)q^2\,, \tag{8}$$

so that one has $x/\xi = (\hat{u} - \hat{s})/q^2$. To leading order (LO) in α_s, the kernel reads

$$C^{q[\sigma]}(\omega) \propto \frac{1}{1 - \omega - i\epsilon} - \sigma \frac{1}{1 + \omega - i\epsilon}\,, \qquad \mathrm{Im}\, C^{q[\sigma]}(\omega) \propto \pi\, [\delta(\omega - 1) - \sigma\delta(\omega + 1)] \tag{9}$$

for both DVCS and meson production. At higher orders in α_s one finds branch cuts in the $\hat{s}$ and $\hat{u}$ channels for $\omega > 1$ and $\omega < -1$, respectively. For the dispersion relations we

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need to know the behavior of the kernels when $|\omega| \to \infty$. The NLO kernels for DVCS can be found in [4], and those for meson production in [5]. For negative signature, one finds $C^{q[-]}(\omega) \sim \omega^{-1}$ up to logarithms for both DVCS and meson production. For positive signature, the NLO corrections give $C^{q[+]}(\omega) \sim \omega^{-1}$ for DVCS, and $C^{q[+]}(\omega) \sim \omega^0$ for meson production, again up to logarithms. The power behavior as ω^0 is due to two-gluon exchange in the t-channel. For DVCS such graphs only start at NNLO, so that at this level one will also have $C^{q[+]}(\omega) \sim \omega^0$. For both signatures one can thus write down an unsubtracted dispersion relation for the kernel,

$$\operatorname{Re} C^{q[\sigma]}\left(\frac{x}{\xi}\right) = \frac{1}{\pi} \int_1^\infty d\omega \operatorname{Im} C^{q[\sigma]}(\omega) \left[\frac{1}{\omega - x/\xi} - \sigma \frac{1}{\omega + x/\xi}\right]. \tag{10}$$

On the other hand, the invariant amplitude satisfies its own fixed-t dispersion relation (3). Therefore the real part of the leading invariant amplitudes for DVCS or meson production can be obtained from a dispersion relation for the hard-scattering kernel,

$$\operatorname{Re} \mathcal{F}^{q[\sigma]}(\xi, t) = \frac{1}{\pi} \int_1^\infty d\omega \operatorname{Im} C^{q[\sigma]}(\omega) \int_{-1}^1 dx \, F^q(x, \xi, t) \left[\frac{1}{\omega\xi - x} - \sigma \frac{1}{\omega\xi + x}\right], \tag{11}$$

or for the invariant amplitude itself,

$$\operatorname{Re} \mathcal{F}^{q[\sigma]}(\xi, t) = \frac{1}{\pi} \int_1^\infty d\omega \operatorname{Im} C^{q[\sigma]}(\omega) \left\{ \int_{-1}^1 dx \, F^q\left(x, \frac{x}{\omega}, t\right) \left[\frac{1}{\omega\xi - x} - \sigma \frac{1}{\omega\xi + x}\right] \right.$$
$$\left. + \mathcal{I}^{q[\sigma]}(\omega, \xi_0, t) \right\}, \tag{12}$$

where ξ_0 corresponds to the subtraction point ν_0 in (3) and

$$\mathcal{I}^{q[\sigma]}(\omega, \xi, t) = \int_{-1}^1 dx \left[F^q(x, \xi, t) - F^q\left(x, \frac{x}{\omega}, t\right)\right] \left[\frac{1}{\omega\xi - x} - \sigma \frac{1}{\omega\xi + x}\right]. \tag{13}$$

As shown in [2], the term $\mathcal{I}^{q[\sigma]}$ is related with spin-zero exchange in the t-channel.

Consistency of the two representations provides nontrivial constraints on the GPDs. Indeed, in (12) the GPD enters in the DGLAP region only, whereas in (11) both the DGLAP and ERBL regions contribute. Let us see that the consistency is guaranteed by the polynomiality property of Mellin moments, which follows directly from the Lorentz covariance of the operator matrix elements that are parameterized by GPDs. With the conventional definitions (given e.g. in [6]) we have for quarks

$$\int_{-1}^1 dx \, x^{n-1} H^q(x, \xi, t) = \sum_{k=0}^{n-1} (2\xi)^k A_{n,k}^q(t) + (2\xi)^n C_n^q(t), \tag{14}$$

$$\int_{-1}^1 dx \, x^{n-1} E^q(x, \xi, t) = \sum_{k=0}^{n-1} (2\xi)^k B_{n,k}^q(t) - (2\xi)^n C_n^q(t), \tag{15}$$

where k is even because of time reversal invariance. Clearly, (11) and (12) are consistent if $\mathcal{I}^{q[\sigma]}(\omega, \xi, t)$ is independent of ξ for all $\omega \geq 1$. To show that this is the case, we Taylor

expand $F^q(x, x/\omega, t)$ in its second argument,

$$\mathcal{I}^{q[\sigma]}(\omega, \xi, t) = \frac{1}{\omega} \sum_{n=1}^{\infty} \frac{1}{n!} \left(\frac{\partial}{\partial \eta}\right)^n \int_{-1}^{1} dx \left(\frac{x}{\omega} - \xi\right)^{n-1} F^q(x, \eta, t) \Big|_{\eta=\xi}$$

$$+ \frac{\sigma}{\omega} \sum_{n=1}^{\infty} \frac{1}{n!} \left(\frac{\partial}{\partial \eta}\right)^n \int_{-1}^{1} dx \left(\frac{x}{\omega} + \xi\right)^{n-1} F^q(x, \eta, t) \Big|_{\eta=-\xi}, \qquad (16)$$

where we have interchanged the order of differentiation and integration. For definiteness let us consider the case $F^q = H^q$. Using the polynomiality property (14) and the fact that C_n^q is only nonzero for even n, we find

$$\mathcal{I}^{q[+]}(\omega, \xi, t) = 2 \sum_{n=2}^{\infty} \left(\frac{2}{\omega}\right)^n C_n^q(t), \qquad \mathcal{I}^{q[-]}(\omega, \xi, t) = 0, \qquad (17)$$

which is independent of ξ as required. In the case $F^q = E^q$ there is an additional minus sign on the r.h.s. of (17), in accordance with (15).

The dispersion representations discussed here can provide a practical check for GPD models in which Lorentz invariance is not exactly satisfied. In particular, we find that even for small ξ the model proposed in [7] leads to serious conflicts with dispersion relations when it is used for calculating the real part of scattering amplitudes [2].

The representation (12) has important consequences on the information about GPDs that can be extracted from DVCS and meson production. To leading approximation in α_s, the imaginary part of the amplitude is only sensitive to the distributions at $x = \xi$, and the only additional information contained in the real part is a constant associated with pure spin-zero exchange, given by (17) at $\omega = 1$. In [1] this was referred to as a holographic property. Beyond leading order, the evaluation of both imaginary and real parts of the amplitude involves the full DGLAP region $|x| \geq \xi$. In addition, the real part depends on the appropriate spin-zero term at all $\omega \geq 1$.

Consider now the comparison of a given model or parameterization of GPDs with data on DVCS or meson production. In a leading-order analysis (which should of course always be restricted to kinematics where the LO approximation is adequate) it is sufficient to characterize each GPD by its values at $x = \xi$, supplemented by a constant for the spin-zero exchange contribution discussed above. On one hand this can be a welcome simplification, and on the other hand it indicates the limitations of an LO analysis: when confronting data with a given GPD one is sensitive to $x \neq \xi$ (and to the details of the spin-zero exchange contribution) only at NLO or higher accuracy.

References

[1] O.V. Teryaev, hep-ph/0510031;
I.V. Anikin and O.V. Teryaev, Phys. Rev. D **76** (2007) 056007 [arXiv:0704.2185 [hep-ph]].

[2] M. Diehl and D.Yu. Ivanov, Eur. Phys. J. C **52** (2007) 919 [arXiv:0707.0351 [hep-ph]].

[3] A.V. Belitsky, D. Müller and A. Kirchner, Nucl. Phys. **B629** (2002) 323 [hep-ph/0112108].

[4] A.V. Belitsky and A.V. Radyushkin, Phys. Rept. **418** (2005) 1 [hep-ph/0504030].

[5] D.Yu. Ivanov, L. Szymanowski and G. Krasnikov, JETP Lett. **80** (2004) 226 [hep-ph/0407207];
N. Warkentin, M. Diehl, D.Yu. Ivanov and A. Schäfer, Eur. Phys. J. A **32** (2007) 273 [hep-ph/0703148].

[6] M. Diehl, Phys. Rept. **388** (2003) 41 [hep-ph/0307382].

[7] A. Freund, M. McDermott and M. Strikman, Phys. Rev. D **67** (2003) 036001 [hep-ph/0208160].